Level of Agreement of Damped Harmonic Oscillator Model with the Experimental Pendulum Setup

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Abstract: This report investigates the correlation between theoretical predictions and experimental results of a damped harmonic oscillator using a self-build pendulum setup. The principal objectives are investigating the pendulum's period dependency or independency on both amplitude and string length, calculating the rate of energy loss quantified by the Q factor using two distinct methodologies, and finding any correlations between Q factor and pendulum string length. Results on the amplitude range where the period is considered as constant, the alignments and disagreements between Q factors calculation from two methods on different string length, and the impact of string length on pendulum's period are demonstrated and analyzed. The experimental results, though somewhat aligned with theoretical predictions from damped harmonic motion equations, exhibit notable discrepancies which are discussed in terms of experimental setup limitations and measurement uncertainties with mathematical reasoning.

1. Introduction

Pendulum is a simple system consist of a heavy mass suspended at the end of a freely rotating spring. We can model the damped harmonic motion of the pendulum using equation [1],

$$\theta(t) = \theta_0 \, e^{\frac{-t}{\tau}} \cos\left(2\pi \frac{t}{T} + \varphi_0\right) \tag{1}$$

where θ_0 is initial amplitude, t is time, φ_0 is phase constant, T and τ are constants that are dependent on pendulum setup. The familiar version of above equation is $\theta(t) = \theta_0 \cos(\omega t + \varphi_0)$,where angular speed ω equals [2]:

$$\omega = \frac{2\pi}{T} \tag{2}$$

One important characteristic of damped harmonic oscillator is its period is amplitude independent for small angles. In this report, we will investigate the amplitude independent threshold; using two methods to calculate the rate of energy lost – Q factor and compare if methods agree with one another; then determine length dependency of period. The goal is to conclude whether the provided theoretical equations and predictions of pendulum characteristics match the experimental results.

1.1 Equations for Q Factor Calculation

Q factor can be calculated using the approximate oscillations required for a pendulum system to decay to $e^{-2\pi}$, or 0.2% of its original energy by 2 methods: finding τ through applying mathematical model using equation (1) and counting oscillations. As we plot the period versus amplitude data, it reveals an exponential decaying sinusoidal pattern. Therefore, to fit the practical experiment, we need to introduce the envelope function [1],

$$A = \theta_0 \, e^{\frac{-t}{\tau}} \tag{3}$$

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which describes how amplitude decays with time. It is a smooth curve connecting the maximum amplitude providing by $\theta(t)$ in equation (1). A more general form to describe the trend of changes in amplitude is:

$$A = a\, e^{bt} \tag{4}$$

leaving $\tau = \frac{-1}{b}$. Using another equation [3]:

$$Q = \frac{\tau\omega_0}{2} \tag{5}$$

we can calculate the Q factor numerically by fitting data points in Python program (see Appendix) to find constant *b* value.

In addition, decay in amplitude in one cycle can describe as $e^{-(\frac{\pi}{Q})}$ [3]. After $\frac{Q}{\pi}$ cycles, the amplitude will decay to $\frac{1}{e}$. If amplitude decays to 46%:

$$\frac{x}{e} = 0.46 \;\; \rightarrow \;\; x \approx 1.25 \tag{6}$$

$$N = \frac{Q}{1.25\pi} = \frac{Q}{x\pi}$$

we can then count number of oscillations *N* to derive Q factor.

1.2 Period Dependency on Amplitude

Theoretically, any unknown relationships could be modeled as polynomial:

$$T = T_0(1 + B\theta_0 + C\theta_0^2 + \cdots) \tag{7}$$

by increasing the number of terms. Plotting data into Python program will give values of B, C … and their corresponding uncertainties. When constant terms are less than double of their uncertainties, $T \approx T_0$. From experiment, period is amplitude independent only within range of $-20 \sim 30\ \pm\ 1°$.

1.3 Period Dependency on Length

The theoretical period could be described using equation [4]:

$$T = 2\pi\sqrt{\frac{L}{g}} \approx 2\sqrt{L} \tag{8}$$

In general, it is a power relationship:

$$T = kL^n \tag{9}$$

where $k = 2.0\ s^2m^{-n}$and $n = 0.5$. This relationship could be further verified by taking log on both sides, which gives slope n and y intercept $\ln(k)$:

$$\ln(T) = \ln(k) + n\ln(L) \tag{10}$$

When released angle is smaller than amplitude independent threshold, experiment does agree with above equations. However, equation (8) returns greater derivations than actual period as amplitude increases.

1.4 Q Factor Dependency on Length

Through controlling the released amplitude and only changing the spring length, list of Q factors was calculated using two methods. Results were plotted into 6 general equations and turns out only modelling Q factors have a power law relationship with length, while Q from number of oscillations have little association to any possible relationships. Therefore, two methods do not agree with each other either due to human errors discussed in section: Uncertainties in Data Collection, or different representations of changes in τ discussed in: Dependency of Q Factor on Length, or both. Q factor does not reveal any other strong association with length except indicating as length increases, Q factor decreases.

2. Experimental Method

The original pendulum setup is shown in figure 1. A sewing thread with mass smaller than 0.2g was connected to a nail drilled into the wood to a 52.6 ± 0.5g cylindrical object. Figure 2 indicates object dimensions.

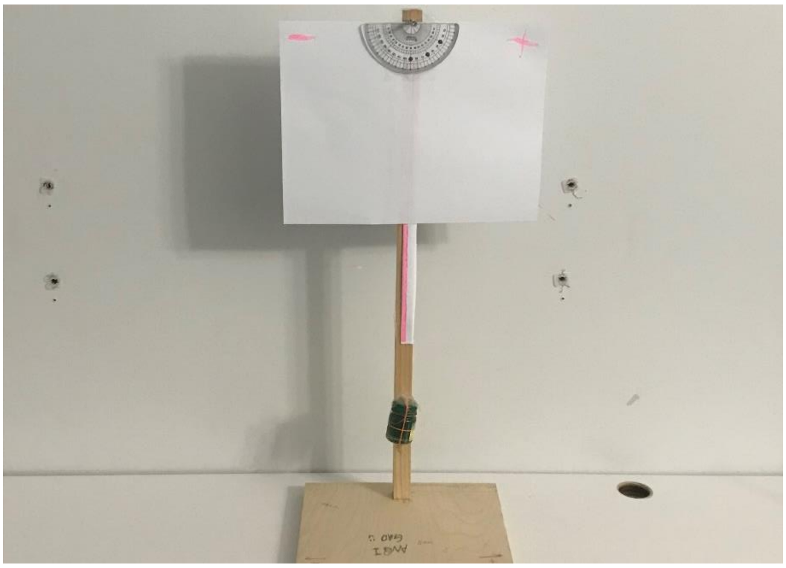

Fig 1. Pendulum setup

A 1080p HD at 30fps iPhone 7 Plus camera was used to track motion, which was placed 60 ± 1cm in front of setup. A straight line marked with pink highlighter acted as a reference to identify if mass has passed central line for data recording. String was hanged 1.9 ± 0.1 cm away from wood to leave just enough space for the travelling object. A light source is placed at the front of the pendulum to create shadow of string on paper. The shadow is used to determine whether the recording device is placed right at the central line, and the accurate angle measurement is based on using the shadow projected on paper to find a parallel line on the protractor**.** The position and angle of the pendulum was obtained by using the *Tracker* software [5]. The raw data collected was then processed in the provided Python program to find the best correlation to describe the dependency of period on amplitude and length and its relationship with Q factor of a pendulum system.

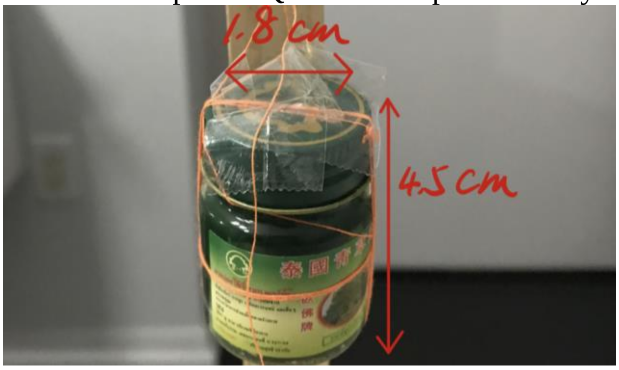

Fig 2. Dimensions of the object being used. Note dimension is not in-scale due to camera angle

However, the potential experimental error caused by the original setup was identified and discussed in section: Experimental Setup Deficiency. To reduce these uncertainties, a new setup was established as shown in figure 3. Another wooden stick was attached to the platform with a pinned nail to the same height as the nail on top of the old wooden stick. Two strings are attached on object and tied to two nails to eliminate self rotation and force the object to travel in one 2D plane. The width of spring has adjusted to $0.5 \pm 0.1mm$ to increase its visibility in recorded videos. The experimental environment is explicitly set with little air turbulence and minimized natural light sources to reduce other variations.

Fig 3. New modified pendulum setup

The length from mass centre to nails is fixed by taping springs at back of sticks. To change the length, it requires pulling the strings manually until the object reach desired height and retape the strings.

Angle and length uncertainty for the first determining amplitude dependency lab, ± 1° and ± 1 mm has modified to ± 4° and ± 4 for the second determining length dependency lab due to changes in pendulum length. For rigorous conclusion, first lab results will write with first set of uncertainty and second lab results will use the second set. Justification for uncertainty changes is in section: Uncertainty in Data Collection and round up to 1 significant digit.

3. Procedure and Results

3.1 Period Dependency on Amplitude

First experiment determines the period from different released amplitude from both positive and negative range, starting from 10 ± 1° with an increment of 10° each up to 80 ± 1°, similar for negative angles. The period was calculated by recording the time elapsed for 5 oscillations, starting and ending in central 0° line then divided by 5. The primary data shown in figure 4 reveals an even polynomial relationship.

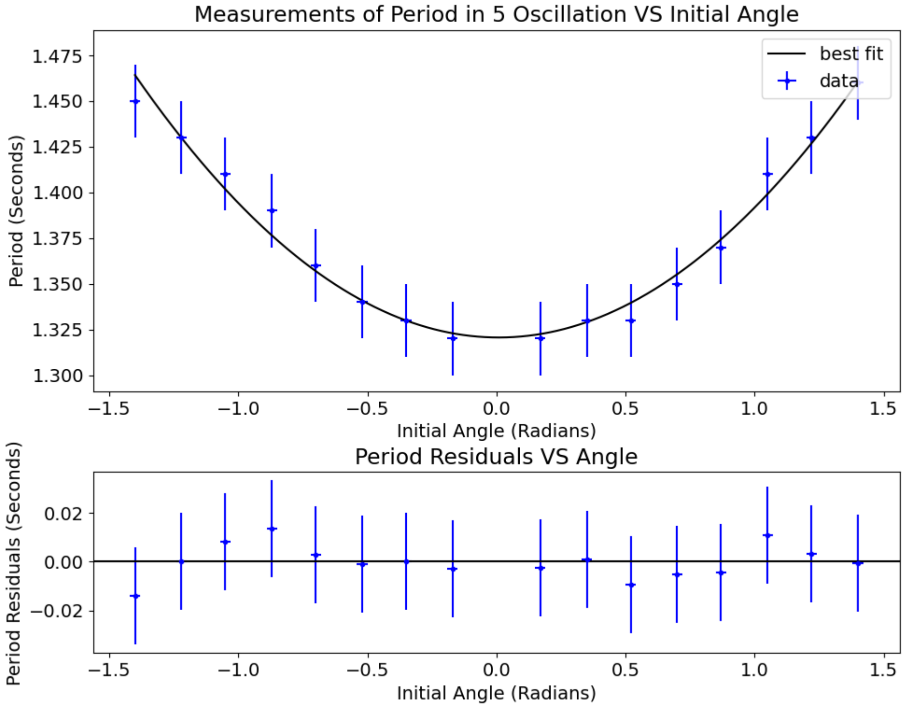

Fig 4. Period measured per 5 oscillations in terms of initial released amplitude and residual graph of period fit into quadratic relation. Uncertainties show in blue line

The residual graph also indicates a relative symmetric pattern, with the greatest difference of 0.02s in period for each pair of corresponding positive and negative angles. Since the period uncertainty is also 0.02s justified in section: Uncertainties in Data Collection, the deviation of period from different sides of pendulum is negligible, and we can claim the pendulum is symmetrical.

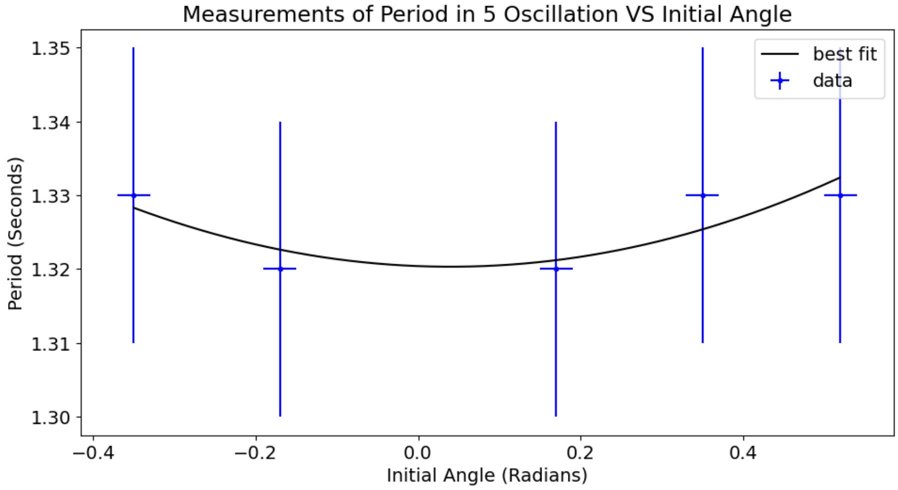

Fig 5. Range of amplitudes when period is constant

By removing data points in figure 4 with greatest amplitude which are extreme data that do not agree with the prediction that period is independent from amplitude, Python program generates figure 5 by building model using equation (7). The calculated constants $B = -0.003 \pm 0.005$ and $C = 0.04 \pm 0.02$. As B is smaller than its uncertainty, and C is up to two times larger than its uncertainty, we can claim B is experimentally 0 and C is consistent with 0. Therefore, within the range of -20 ~ 30 ± 1°, period is unaffected by amplitude.

3.2 Q Factor Calculation

The second experiment was conducted to calculate Q factor using two methods. Through recording a full oscillation started from 40 ± 1° and measuring the maximum amplitude and time elapsed for each 10 cycles, as shown in figure 6, equation (4) could model the relationship between maximum amplitude and time.

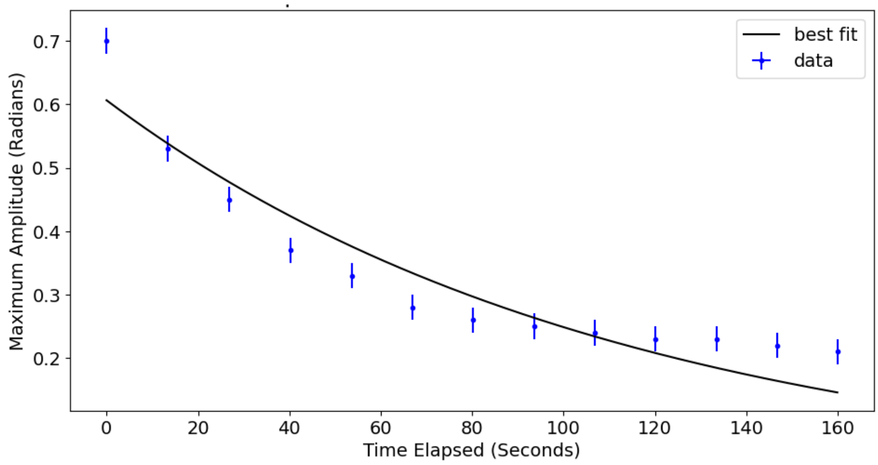

Fig 6. Measurements of maximum amplitude vs time elapsed. An exponential decay pattern is observed

Plotting the data in Python gives value of b in equation (4), which can use to calculate τ thus Q using equation (5). The value of period is selected as the one unaffected by amplitude calculated from figure 5, which gives Q = 267 ± 30. Using the second method: count number of oscillations, the measured Q = 417 ± 100 by recording data from amplitude of 18.4 ~ 9.0 ± 1° , which decays 48.9%

for 95 oscillations as shown in figure 7. Using equation (6), applying the average calculation and follow the largest uncertainty percentage rule, the final Q factor has:

$$Q = 342 \pm 60$$

Two calculated Q factors only overlap 2% accounting uncertainty, which means they are highly unagreed with each other. One possible hypothesis is the differences in released amplitude. Since the first method has a greater starting degree, it releases more energy with larger decaying rate during the first several oscillations, which lower the average Q factor as it takes fewer oscillations to come to a complete stop. The second method has a much smaller initial amplitude, therefore the rate of energy lost is also small.

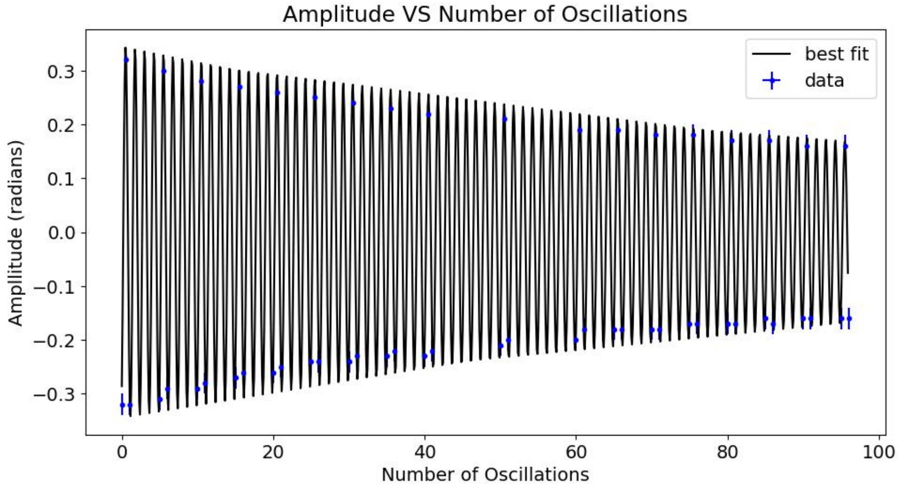

Fig 7. Data are recorded for each 5 oscillations at maximum amplitude; fit in a sinusoidal relation

In addition, τ modelled in figure 6 and in figure 7 has $\Delta\tau \approx 21s$, which gives $\Delta Q \approx 50$ that falls in the uncertainty range of final Q value. Therefore, we may conclude that even if two Q values deviates much based on different method of measurements, the resultant Q value is still relatively consistent with data considering the different released amplitude and the nature of decaying energy trend.

However, in section: Length Dependency on Q Factor, with controlled released amplitude and pendulum length, Q factor in two methods demonstrates unusual correlation. It suggests the complexity of Q is being influenced by multiple factors and any slight changes to one factor may result in disagreement of Q value for two methods. Detailed discussion is written in that section.

In general, higher Q value is preferred as it represents a more theoretical model of pendulum that have a constant rate of energy lost. The deviations in two Q values may result from a combination of mistaken choices of experimental design, limited methods to reduce uncertainties and insufficient data collection. All these possibilities will be examined in the whole section: Analysis and Discussion to derive a final statement of whether two methods are consistent with each other.

3.3 Period Dependency on Length

All 15 trials from length 48.3 to 14.4 cm for determining the length dependency of period are less than 30 ± 4°, which was determined to be the maximum amplitude threshold so that period is considered unaffected by amplitude.

Fitting the measured period and length into equation (9) as shown in figure 8, the power law gives slope k and intercept n as:

$$k = 2.01 \pm 0.04\ s^2 m^{-n}$$

$$n = 0.55 \pm 0.02$$

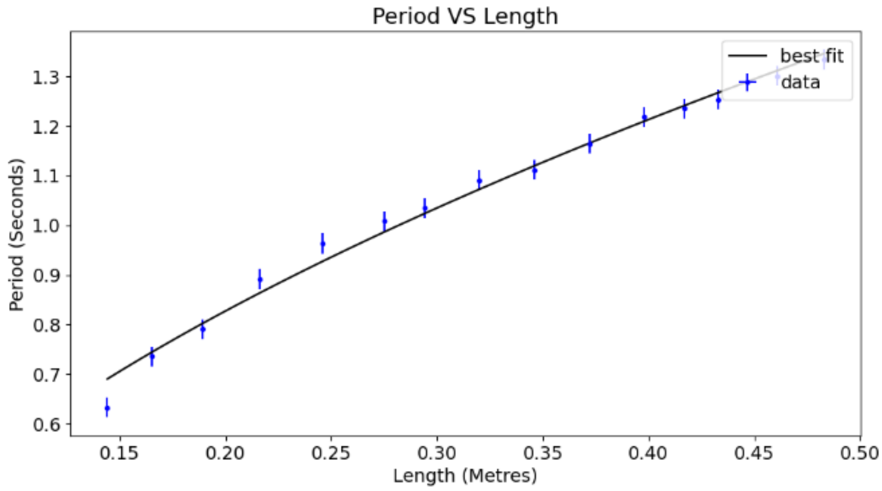

Fig 8. Data collected from length of 48.3 to 14.4cm fit in power relationship.

By inspecting equation (8), which gives the theoretical period that only works for small amplitude, it gives slope k and intercept n as:

$$k \approx 2.0\ s^2 m^{-n}$$

$$n \approx 0.5$$

The maximum deviations of k is $1.5\% - 2.5\%$ and that of n is $6.0\% - 14\%$ using the extremes of actual k and n value. Even if the percentage difference for n is relatively large to be considered the same, I still claim equation (9) is consistent with equation (8) because the released amplitude is just below the threshold where the period is unaffected, and the angle may exceed that threshold due to human error.

To verify the claim, log-log graph in figure 9 is also plotted by manually calculating the natural log of length and period. The new equation (10) indicates the slope becomes n and y intercept is $\ln(k)$ in figure 9. Performing some backward calculations give:

$$k = 2.05 \pm 0.03\ s^2 m^{-n}$$

$$n = 0.57 \pm 0.02$$

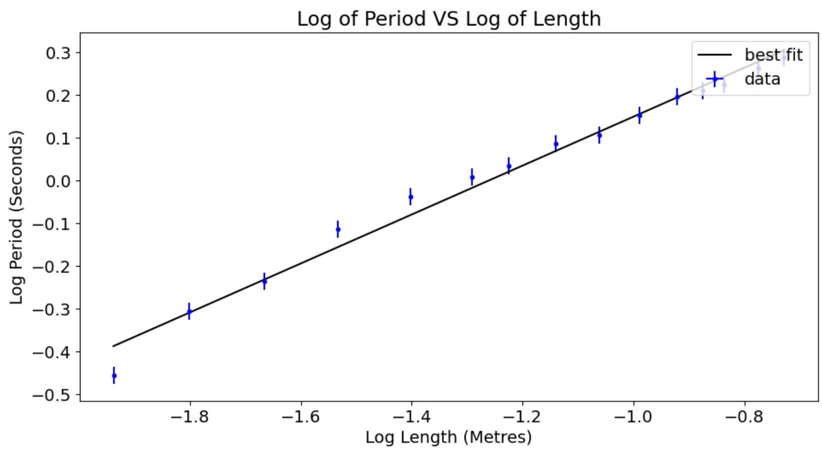

Fig 9. Applying ln to figure 8 to get log-log graph

The maximum variations of k and n in this case is 4% and 14% respectively. Therefore, it also roughly agrees with the theoretical model.

3.4 Q Factor Dependency on Length

Methods of modelling pendulum motion and counting number of oscillations are both used to calculate Q factor even if only one is required. Considering the average of 150 units variations in Q factor calculated with different methods in section: Q Factor Calculation, without logical reasonings for this great difference, there is no convincing justification to prefer one method over the other.

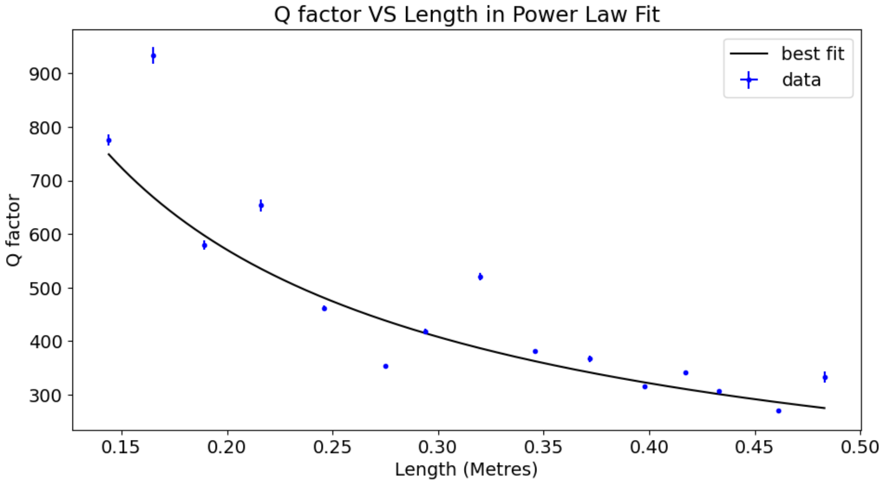

Fig 10. Q factor from modelling method in terms of length fit in power law function

Results from both methods are fit into rational function $Q = \frac{a}{L-h} + k$, quadratic function $Q = aL^2 + bL + c$, power law function $Q = kL^n$, linear function $Q = mL + b$, exponential function $Q = ae^{bL}$, and polynomial function $Q = a(1 + bL + cL^2 + dL^3)$ (see Appendix). The best relationship for modelling method is power law, given:

$$k = 150 \pm 20\ s^2 m^{-n}$$

$$n = 0.83 \pm 0.1$$

$$Uncertainty\ Percentage\ k = 12\%$$

$$Uncertainty\ Percentage\ n = 12\%$$

as shown in figure 10**.** Uncertainty percentage is the ratio of the actual values over uncertainties generated from Python program times 100. The best relationship for the counting method, however, is difficult to select between linear and power law indicated in figure 11 and 12 as linear relationship has:

$$Uncertainty\ Percentage\ m = 42\%$$

$$Uncertainty\ Percentage\ b = 9.7\%$$

while that for power law relationship is:

$$Uncertainty\ Percentage\ k = 12\%$$

$$Uncertainty\ Percentage\ n = 36\%$$

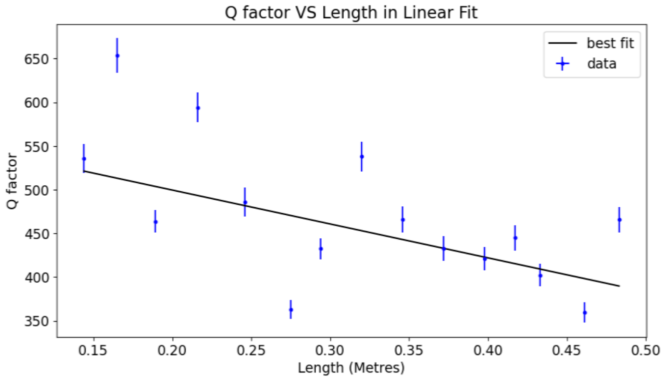

Fig 11. Q factor from counting method in terms of length fit in linear function

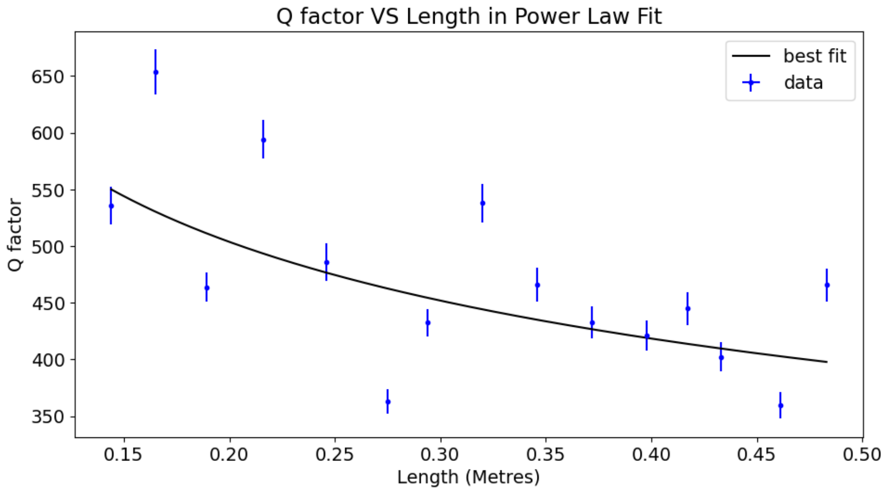

Fig 12. Q factor from counting method in terms of length fit in power law function

For experimental accuracy, I claim that none of the tested relationships could fit the data derived from counting method, as the greatest uncertainty for constants *m* and *n* is almost half of the actual value. For the modelling data, power law *may be* used to determine Q factor dependency which also match the theory if we modified equation (5) into the power law form:

$$Q = \frac{\pi\tau}{2} \times L^{\frac{-1}{2}} \tag{11}$$

by substituting equations (2) and (8).

It is mathematically supported that the modelling method gives a more reliable estimate of Q factor dependency on length due to smaller uncertainties. Counting method on the other hand produces a weak negative association that is also under expectation since the initial released and final amplitude measurements are purely based on raw eyes without any precise instruments, which is an extremely critical flaw for using this method as the calculation only requires one difference in initial and final amplitude.

4. Analysis and Discussion

4.1 Uncertainties in Data Collection

All uncertainties for raw data collection that are not obtained by *Tracker* are determined by half of the most accurate digit that can measure from the measuring devices and will be automatically round to 1 significant digit. Time uncertainty is practically determined in 30fps even though the device could record up to 60fps. The reason is half of the uncertainty differences between 60 and 30 fps, $\Delta t \approx 0.02s$, is negligible when we divided by total number of oscillations. In this case using 95 oscillations from figure 7, we get $\frac{\Delta t}{95} \approx 2.1 \times 10^{-4}s$. Therefore, we could process data in 30fps without missing any time uncertainty.

The distance to mass centre also presents uncertainty, giving the top quarter of the object is its cap with mass $\approx 7.6 \pm 0.5g$, which weighed much less than the body. Performing some basic ratio calculations involving object height, the theoretical mass centre should be 2.3cm below the top of object cap, while the actual measured distance is 2.5cm. Using the equation (8) to estimate period, it results $\Delta T \approx 0.03s$, which has the same magnitude as the time uncertainty. Since period is proportional to Q factor, the relative variations in Q will be similar to that in period.

Another type of uncertainty in length revealed after the modification of experimental setup, which is caused by non-parallel surface of object to platform due to imperfection of human eyes in tying two strings to same height. In reality, one string must be less than the other, causing the mass centre to shift location as shown in figure 13**.** Since the mean difference threshold for stimulus length under nonsimultaneous condition is 2.64mm [6], the assumption that eyes can only detect maximum 3mm length difference is acceptable. Another assumption is made that the mass centre will shift exactly in the same direction as the longer spring with the same magnitude equal to the exceeded length. Hence, the estimated uncertainty is $\pm 3mm$. Add the $\pm 1mm$ from accuracy of length measurement, the actual uncertainty in length should be $\pm 4m$.

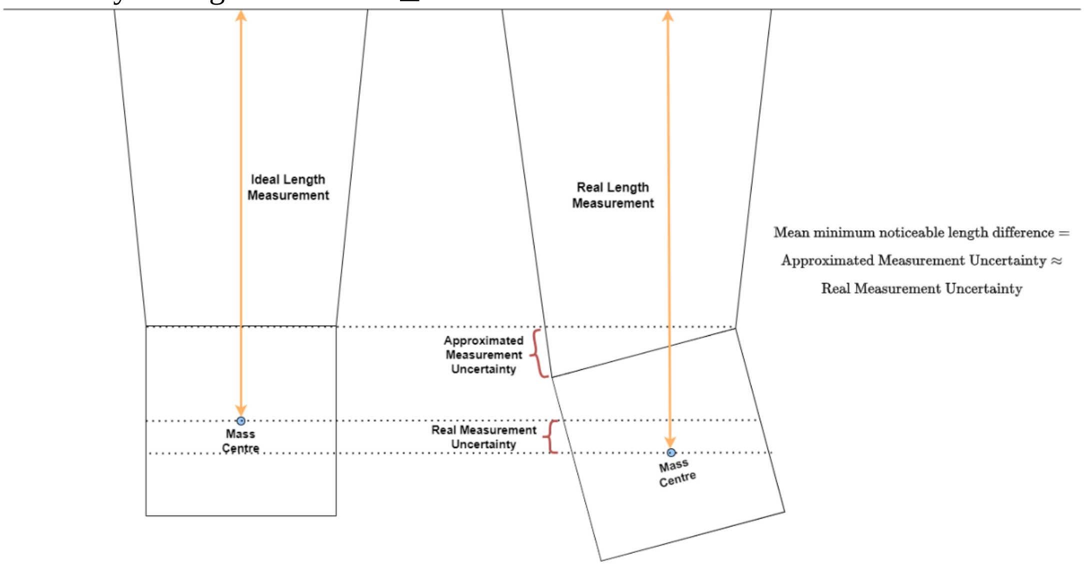

Fig 13. Visual representation of length uncertainty for non-parallel object surface

Even if the experimental setup and environment has been modified, the ambiguity of the location of the mass in recorded video is still unavoidable, which results in angle uncertainty. The estimated location variation is set as the largest range of possible uncertainty, the radius of object, $\Delta x \approx 0.9cm$. Using the basic trigonometry and distance to mass centre in lab 1, we calculate the angle uncertainty $\Delta\theta \approx 1.2°$. While this uncertainty is acceptable for greater released angle, it becomes a serious issue while collecting sample data to plot figure 7. As the amplitude interval is only 9.4°, angle uncertainty is almost 13% of the whole interval.

For further experiments in determining length dependency, angle uncertainty varies as distance from mass centre changes, with maximum $\pm 4°$ to minimum $\pm 1°$ calculated based on the longest and shortest length $48.3cm$ and $14.4cm$ respectively. Using a sample data from figure 10 and recalculate Q using counting method including uncertainty, the range of Q factor uncertainties is $\pm 30 \sim \pm 90$, which on average is $\approx 6\% \sim 18\%$ to the actual Q. The shorter the distance, the greater the angle uncertainty thus greater error in Q factor calculation. There is currently no direct solution to minimize the angle uncertainties without technical equipment other than smartphone camera.

4.2 Experimental Setup Deficiency

There are three major flaws in the original experimental setup. First, even though the object is set very close to the standing wood, it does not prevent 3D oscillations, which is extremely critical to measurement accuracy. The object may hit the wood within 15 oscillations if the initial amplitude is above $20 \pm 1°$ for careless release. It is also the major reason of choosing $18.4 \pm 1°$ in section: Q Factor Calculation using counting oscillations method to find Q value instead of matching the $40 \pm 1°$ for modelling method.

The second flaw is the setup is too sensitive to perturbations that the air flow from air conditioner will cause the object keeps rotating for majority of the time. Although the implications and degree of influence of this additional rotational motion is uncertain, it could possibly lead to other complex types of faults. Another evidence is the preference of positive amplitude over negative for constant period threshold in figure 5. Since warm air is blowing from positive to negative side in original setup, there is a possibility that for the first several oscillations, the air contributes to the swing that increase the tangential speed and thus reduce the period from positive to negative direction and vice versa. Therefore, the variance of period in positive amplitude will decrease and that in negative will increase. Solution to both flaws was implemented and described in section: Experimental Method. Additionally, avoid any extra flowing air that may increase the complexity of the system.

The third flaw is although it is better to use lighter spring to reduce effect of spring mass on pendulum system, the width of spring should not be less than 0.2 ± 0.1 mm, which is the original spring width used in experiment. When the light is projecting on spring, sometimes the shadow is barely seen due to interference of artificial light source and natural light source which cause continuous adjustments in camera light exposure. The higher the exposure value, the harder for eyes to identify shadows. We can either replace the string with a thicker one or perform the experiment at night to reduce natural light source to minimum, which were applied in the modified setup.

The fourth flaw in design raised up after the setup modification due to that extra added wooden stick, which is not securely attached to the platform, causing the whole pendulum system to shake slightly for first one or two oscillations. Influence of this flaw is not clearly determined but could be estimated. Using the conservation of energy, the ideal system uses all potential energy to keep the object swinging while the actual system required some energy to resist vibration of wooden stick, leaving less energy remaining for object oscillations. Thus, the measured Q factor would decrease assuming the damping ratio (τ) is the same. To fix this issue, we can simply replace the stick with a new one and skewing it tightly to the platform.

4.3 Period Dependency on Amplitude

While the experimental setup certainly has other flaws that require improvements, all experiment results clearly imply the nature of period is non-linear and is dependent on amplitude as the initial amplitude exceeds certain range, in this case -20 ~ 30 ± 1°. The equation (8) holds true with $\Delta T \approx 0.004 \sim 0.014s$ within the range of -30 ~ 30 ± 1° by comparing the theoretical calculated period and actual modelled period. The variations are experimentally 0 since $\Delta T < 0.02s$, less than the uncertainty for time measurements. However, as the released amplitude increase, theory does not hold true anymore.

As shown in figure 14, the forces exert on mass is mg and T in an oscillating motion. Tension force and vertical component of gravitational force sum up to 0, leaving $-mgsin(\theta)$ the only force exert on mass if treating right direction as positive. Replacing this force in $F = ma$ we get:

$$a_{tan} = -gsin(\theta)$$

If we replace tangential acceleration with angular acceleration using $a_{tan} = r \times \alpha$ [7], we have:

$$\alpha = \frac{-gsin(\theta)}{L}$$

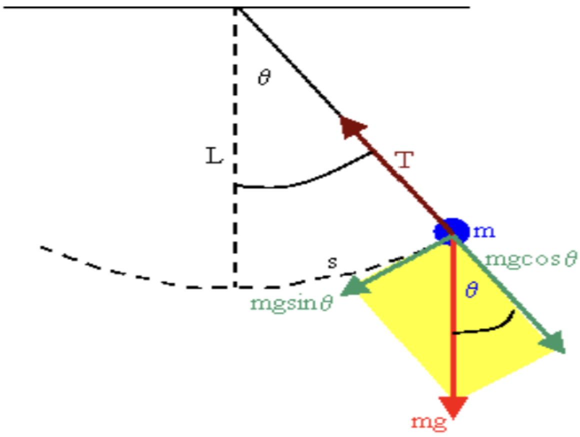

Fig 14. Free body diagram of a swinging mass in pendulum. Image obtained from website [8]

In theory, equation (8) is derived by assuming $sin(\theta) \approx \theta$[9], which gives $\alpha = \frac{-g\theta}{L}$, a linear relationship between α and θ. However, the real α has smaller magnitude when θ exceeds ~0.5rad, as indicated in figure 15.

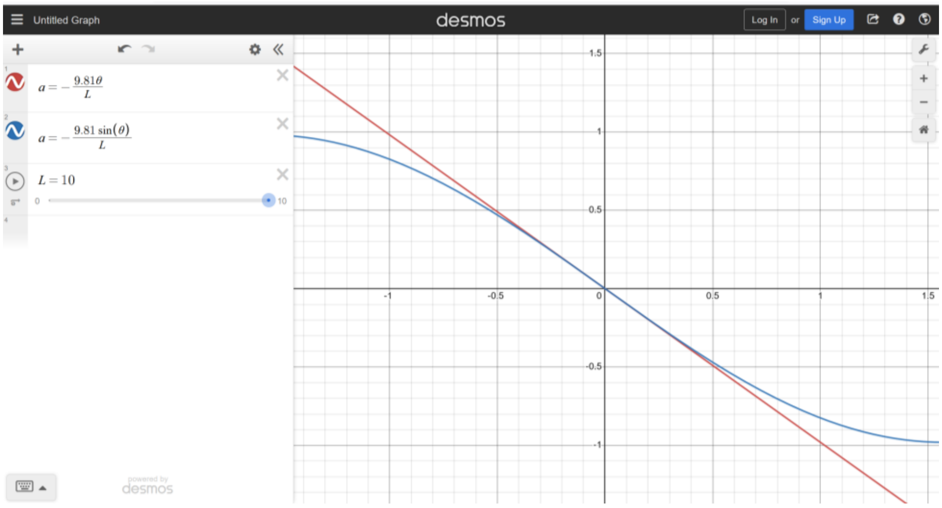

Fig 15. Graphing theoretical and actual α functions in terms of θ using Desmos [10]

If we manually pick the red function in figure 15 as x-axis, the deviations of blue function α in terms of the x-axis is "quadratic-like". Since smaller acceleration results in larger period, and α is proportional to square of angular speed, and speed is also proportional to period in equation (2)**,** it explains the even polynomial relationship between practical period and amplitude in figure 4.

4.4 Q Factor Dependency on Length

The big trend of Q factor vs length is decreasing for both calculating methods, which is difficult to justify systematically but could be explained using theoretical equation (11). Assuming τ is constant, the relationship between Q and L is inversely proportional.

However, τ is not a constant in reality, as it measures the viscosity of the whole pendulum system which is affected by so many factors such as humidity and instantaneous adhesion between surfaces. It is the major cause of the irrelevance between Q and L by inspection in figure 16.

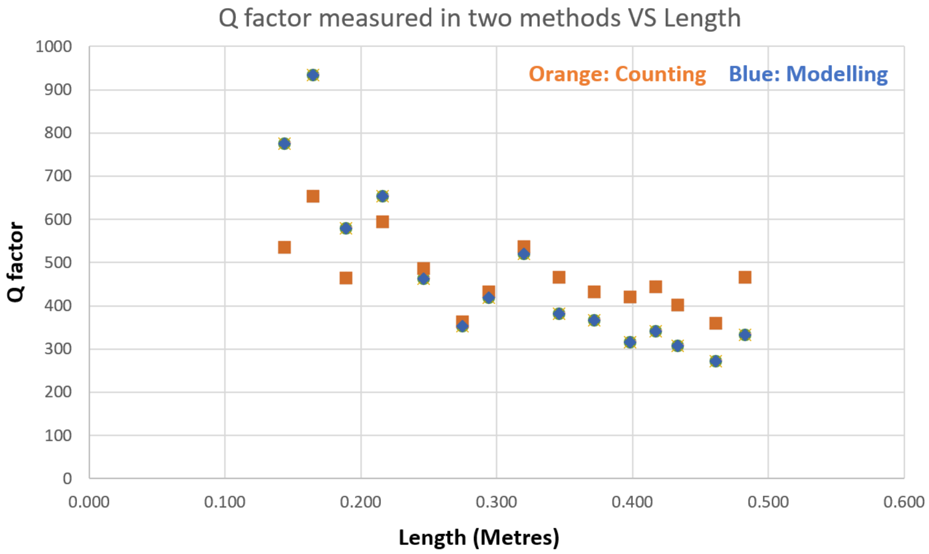

Fig 16. Variations in Q factor calculations from counting and modelling methods

From 21.6cm to 14.4cm, the measured Q factor from counting is less than that from modelling, while from 32cm to 24.6cm two methods highly agree with each other. The difference of Q factor between 34.6cm to 48.3cm is roughly the same from two methods. We can at least conclude from experimental results that the instantaneous τ by modelling in greater length is less than that by counting, in medium length is similar and in shorter length is greater.

There are two major factors required to consider for deviations in τ: Tension Force and Air Resistance. Assuming object is in static motion and experience symmetrical forces, as the length decrease, the tension force will increase since the x components of spring increase due to larger angles while y components remain the same for same weight as shown in figure 17. Given $F_f = \mu N$, greater N (tension force in this case) indicates greater friction force thus more thermal energy is released from the rubbing effect between the connections of spring and nail, decreasing τ thus reducing the actual Q factor.

Since period data is collected under amplitude independence, greater length means further distance to travel per oscillation thus greater wavelength. From general equation $T = \frac{\lambda}{v}$, it is evident that maximum velocity for greater length is larger than that with smaller length. Since air resistance is proportional to velocity squared by theory from drag equation $\overrightarrow{F_{air}} = -\frac{1}{2}\rho AC \left|\overrightarrow{v}\right|^2 \hat{v}$ [11], it is mathematically proved that greater length results in greater air resistance, which does more negative work on the system as the resistance force direction is always opposite to the direction of motion thus decreasing τ and reducing Q factor.

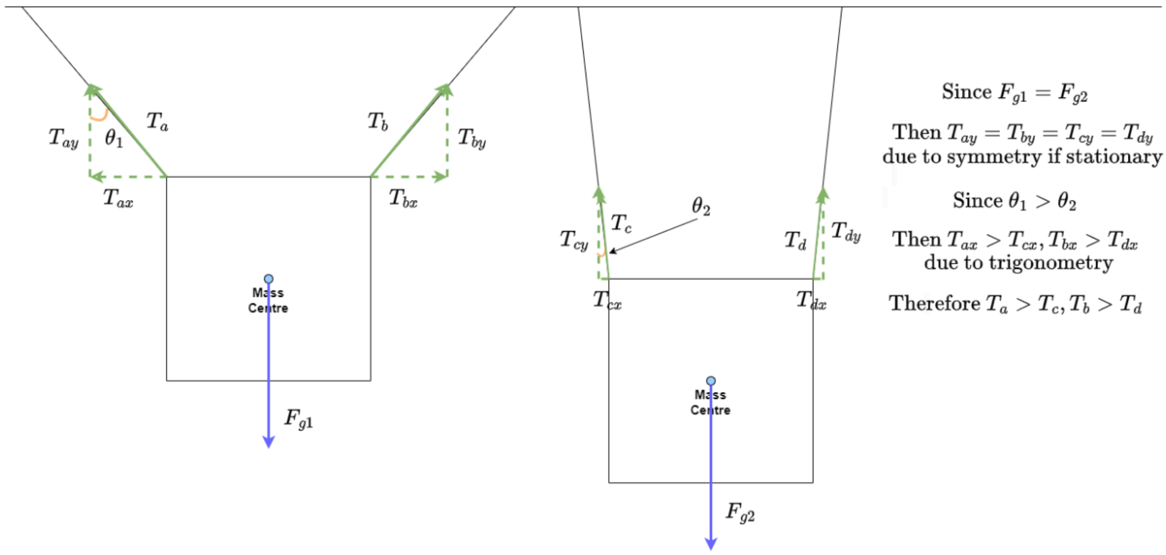

Fig 17. Visual demonstration for greater tension force in shorter length

A hypothesis is that two Q factor calculation methods weigh these two factors differently: counting reflects the reality of tension force while modelling reflects more on the effect of air resistance. However, performing logical justification requires advanced research, therefore a final statement to conclude trends in figure 16 is not provided.

5. Conclusion

In conclusion, it appears the damped harmonic oscillator does not fully describe a swinging pendulum system. Period could only be considered amplitude independent within -20 ~ 30 ± 1°, but the amplitude does decays exponentially so does the energy lost. $T = 2\sqrt{L}$ returns perceptible deviations for greater amplitude. Two Q factor measurements do not agree with each other, providing one could fit into the power law relationship while the other have little association to all 6 tested functions. Q factor does not reveal any strong relationship with length except providing a big image that as length increases, Q factor decreases.

For future experiments, repeated data collections are encouraged to reduce the effect of angle and length measurement variances, which are considered as the largest uncertainty in this lab. It also helps discover and quantify additional factors which may affect Q factor and τ other than length. It would be ideal to record data with a professional measurement instrument and using some systematics methods to determine uncertainty based on raw data.

Acknowledgements and Appendix

The Python program mentioned in the report is available in Quercus, provided by Professor Bentz Wilson for graph generating.

All resources used to find the best fit of relationships for Q factor VS length is available. Please contact the author if raw data is required and will be sent in a folder.

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